UDC 62-50

REDUCTION PROBLEM WITH CONSTRAINTS ON THE TOTAL MOMENTA OF THE CONTROLS

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We examine the reduction problem for the trajectories of a conflict-controlled linear stationary system in a specified neighborhood of the equilibrium position. The actions of the opponents are restricted by constraints on the magnitude of the total momenta of controls. Such constraints admit of stepwise displacements along directions capable of piercing the target set. The considered problem is reduced, by means of the generalized impulse calculus, to an auxiliary one which is solvable by the methods in [1]. The obtained impulse extremal construction depends upon the initial position and admits of stepwise motions in the sliding mode. In the general case it is necessary for the party interested in the reduction to know the part of the trajectory realized up to the instant of making a decision. In Sect. 1 we examine the problem of the encounter of two material points of variable mass, while in Sect. 2 the reduction problem for a multidimensional conflict-controlled stationary system with a nonsingular matrix is considered.

1. The encounter of two material points. Let x be the distance between two points moving along a straight line under the action of reactive forces. The encounter process can be described by the Meshcherskii equation

$$x^{\cdot\cdot} = u - v; \quad u = -c_1\frac{m_1^{\cdot}}{m_1}, \quad v = -c_2\frac{m_2^{\cdot}}{m_2} \tag{1.1}$$

In (1.1) m_i is the mass of the i th point $(i = 1,2)$ and c_i is the relative discharge velocity of the reactive mass. Let t_0 be the time reference point. We assume that the mass $\Delta m_i(t)$ is subject to the constraint

$$|\Delta m_i(t)| \leqslant \Delta m_i, \quad t_0 \leqslant t \tag{1.2}$$
$$\Delta m_i(t) = m_i(t) - m_i(t_0); \quad \Delta m_i(t) = 0, \quad t < t_0$$

Below we assume that the controls u and v are exercised by the first and second players, respectively. Let t_0 and θ $(t_0 < \theta)$ be the instants of beginning and completion of the game. The first player's aim is to realize at the instant θ the inequality

$$x^2 + x^{\cdot 2} \leqslant \varepsilon^2 \tag{1.3}$$

The second player's task is to hinder this. Let us formalize this problem in terms of the differential game theory. To do this it is necessary to ascertain the structure of the constraints imposed on controls u and v by the formulation of the problem. We denote the current value of the total momentum of control u by $\mu(t)$. Then

$$\mu(t)=\int_{t_0}^{t} u\,d\tau = c_1 \ln\frac{m_1(t_0)}{m_1(t)} \approx -c_1\frac{\Delta m_1(t)}{m_1(t_0)}, \quad \Delta m_1 \ll m_1(t_0)$$

Hence the estimate

$$|\mu(t)| \leqslant \mu_0, \qquad \mu_0 = c_1\frac{\Delta m_1}{m_1(t_0)} \tag{1.4}$$

follows from (1.2). The estimate

$$|\nu(t)| \leqslant \nu_0, \qquad \nu_0 = c_2\frac{\Delta m_2}{m_2(t_0)} \tag{1.5}$$

where $\nu(t)$ is the current value of the total momentum of control v is derived analogously. Thus, constraints (1.2) on the reactive masses are equivalent to constraints on the momenta of the corresponding reactive forces.

Below we determine the players' admissible strategies [1]. But first we stipulate the class of admissible strategies M for forming the total momentum $\mu(t)$ of control u. We assume that the strategy M associates a set $M(t, x)$ of segment (1.4) with each position (t, x), $x = (x, x^{\cdot})^T$. Here we assume that the sets $M(t, x)$ are closed and convex with respect to x and are semicontinuous from above with respect to inclusions under the variation of (t, x). Admissible strategies N for forming the total momentum $\nu(t)$ are determined analogously.

The total momentum $\mu(t)$ $(\nu(t))$, locally summable with respect to time and generated by strategy M (respectively, by strategy N), permits us to determine the control

$$u = \mu^{\cdot} \quad (v = \nu^{\cdot}) \tag{1.6}$$

Thus, we can state the problem.

Problem A_1. For a given initial position (t_0, x_0) find a strategy $M^{(H)}$ which for any choice of admissible strategy N ensures that the point $x(t, t_0, x_0, M^{(H)}, N)$ reaches set (1.3) at the instant θ.

We pass on to the construction of an auxiliary reduction problem. For our purposes the first-order generalized impulse calculus proves to be sufficient [2]. In shch a calculus the original z is expanded into the integral

$$z = \int_{-\infty}^{\infty} \delta^{\cdot}(t-\lambda)\, z^*(\lambda)\, d\lambda$$

with respect to shifts of the dipole $\delta^{\cdot}$ (here the integration is understood in the meaning of [2]). The transform z^* can be obtained by a convolution [3] of the original with the Heaviside function $\chi(t - t_0)$ equal to zero for $t < t_0$ and to unity for $t_0 < t$.

Let us write Eq. (1.1) in the form of system

$$\begin{gathered} x_1^{\cdot} = x_2 + x_{10}\,\delta(t-t_0), \quad x_2^{\cdot} = u - v + x_{20}\delta(t-t_0) \\ x_1 = x,\ x_2 = x^{\cdot}, \quad x_{10} = x(t_0), \quad x_{20} = x^{\cdot}(t_0) \end{gathered} \tag{1.7}$$

where δ is the Dirac impulse function which with the specified initial conditions causes the motion of system (1.7). For $t_0 \leqslant t$ we define the shift by

$$y_1 = x_1^*, \qquad y_2 = x_2^* + x_{20} \tag{1.8}$$

In the new coordinates the transorm system has the form

$$y_1^{\cdot} = y_2, \quad y_2^{\cdot} = \mu - \nu + x_{20} \quad , \quad y_1(t_0) = 0, \quad y_2(t_0) = x_{10} \tag{1.9}$$

Using the inverse transform formula $x_i = x_i^{*\cdot}$ $(i = 1, 2)$, the transormation (1.8) and the system (1.9), we can obtain relations connecting the parameters of the original and auxiliary problems $(t_0 \leqslant t)$

$$x_1 = y_2, \quad x_2 = \mu - \nu + x_{20} \tag{1.10}$$

Substituting (1.10) into (1.3) we have for the sought auxiliary problem the target set

$$y_2^2 + [\mu(\theta) - \nu(\theta) + x_{20}]^2 \leqslant \varepsilon^2 \tag{1.11}$$

Thus, Problem A_1 is equivalent to the following problem.

Problem B_1. For a given original position (t_0, y_0), where $y_0 = (0, x_{10})^T$, find the strategy $M^{(H)}$ which for any choice of admissible strategy N ensures that the point $y[t, t_0, y(t_0), M^{(H)}, N]$ reaches set (1.11).

The values of $\mu^{(H)}(\theta)$ constituting the set $M^{(H)}(\theta, y)$ are sought as solutions of the equation

$$\min_{\mu(\theta)} \max_{\nu(\theta)} \{y_2^2(\theta) + [\mu(\theta) - \nu(\theta) + x_{20}]^2\} = \max_{\nu(\theta)} \{y_2^2(\theta) + [\mu^{(H)}(\theta) - \nu(\theta) + x_{20}]^2\}$$

We obtain

$$\mu^{(H)}(\theta) = \begin{cases} \mu_0, & x_{20} < -\mu_0 \\ -x_{20}, & |x_{20}| \leqslant \mu_0 \\ -\mu_0, & \mu_0 < x_{20} \end{cases} \tag{1.12}$$

The best action of the second player is determined by the formula

$$\nu^{(H)}(\theta) = \begin{cases} \nu_0, & x_{20} < -\mu_0 \\ \pm\nu_0, & |x_{20}| \leqslant \mu_0 \\ -\nu_0, & \mu_0 < x_{20} \end{cases} \tag{1.13}$$

Substituting (1.12) and (1.13) into (1.11), we have the pre-target set in Problem B_1

$$|y_2(\theta)| \leqslant h(x_{20}) \tag{1.14}$$

$$h(x_{20}) = \begin{cases} h_- = [\varepsilon^2 - (\mu_0 - \nu_0 + x_{20})^2]^{1/2}, & x_{20} < -\mu_0 \\ h_0 = (\varepsilon^2 - \nu_0^2)^{1/2}, & |x_{20}| \leqslant \mu_0 \\ h_+ = [\varepsilon^2 - (-\mu_0 + \nu_0 + x_{20})^2]^{1/2}, & \mu_0 < x_{20} \end{cases}$$

An analysis of function h yields the necessary reduction condition

$$\nu_0 < \varepsilon \tag{1.15}$$

and the region of initial conditions in Problem A_1, from which reduction is perhaps possible

$$|x_{20}| \leqslant \varepsilon + \mu_0 - \nu_0 \tag{1.16}$$

Thus, to solve Problem B_1 it is sufficient to solve the following problem.

Problem C_1. For a given original position $(t_0, y(t_0))$ find the strategy $M^{(H)}$ which for any choice of admissible strategy N would ensure that the point $y[t, t_0, y(t_0), M^{(H)}, N]$ reaches set (1.14) at the instant θ.

We solve this problem in accordance with [1]. The set $W(\theta, t)$ of program absorption of the target (1.14) is determined as the set of vectors $w = (y_1, y_2)^T$ which satisfy the condition

$$-\int_t^\theta s^T X(\theta, \tau)(0, x_{20})^T d\tau + \rho_2(s, t, \theta) - \rho_1(s, t, \theta) - \rho_{-M}(s) - s^T X(\theta, t) w \leqslant 0 \tag{1.17}$$

for all unit vectors s. In (1.17) X is the fundamental matrix of the homogeneous system (1.19), ρ_{-M} is the support function of set $-M$ (M is set (1.14)), and ρ_i is the support function of the attainability set of the i th player ($i = 1, 2$). We have

$$\rho_1 = \max_{|\mu| \leqslant \mu_0} \int_t^\theta [s_1 (\theta - \tau) + s_2] \mu (\tau) \, d\tau = \tag{1.18}$$

$$\mu_0 \int_t^\theta | s_1 (\theta - \tau) + s_2 | \, d\tau, \quad \rho_2 = \nu_0 \int_t^\theta | s_1 (\theta - \tau) + s_2 | \, d\tau$$

$$\rho_{-M} (s) = \begin{cases} \infty, & s_1 \neq 0 \\ h, & s_1 = 0 \end{cases}$$

Substituting (1.18) into (1.17) and computing the maximum of the left hand part with respect to s, we obtain

$$s^\circ (t, y) = (0, - \operatorname{sign} [x_{20} (\theta - t) + y_2])^T \tag{1.19}$$

$$W (\theta, t) = \{y \mid H_- (t) \leqslant y_2 \leqslant H_+ (t)\}$$

$$H_\pm (t) = \pm h + (\pm \mu_0 \mp \nu_0 - x_{20}) (\theta - t)$$

The set $W (\theta, t)$ is nonempty if

$$0 < h + (\mu_0 - \nu_0) (\theta - t_0) \tag{1.20}$$

We compute the support function

$$\rho_W (s) = \begin{cases} \infty, & s_1 \neq 0 \\ \pm H_\pm, \quad s_2 = \pm 1, & s_1 = 0 \end{cases} \tag{1.21}$$

Allowing for (1.21)

$$\varkappa (t, y) = \max_{\|s\| \leqslant 1} [s^T y - \rho_W (s)] = \begin{cases} y_2 - H_+, & - x_{20} (\theta - t) < y_2 \\ - y_2 + H_-, & y_2 < - x_{20} (\theta - t) \end{cases} \tag{1.22}$$

The strategy which is extremal relative to $W (\theta - t)$, has the form

$$M^{(e)} (t, y) = \begin{cases} \{\mu \mid |\mu| \leqslant \mu_0\}, & \varkappa (t, y) \leqslant 0 \\ M^{(e)} [t, s^\circ (t, y)], & 0 < \varkappa (t, y) \end{cases} \tag{1.23}$$

where

$$M^{(e)} [t, s^\circ (t, y)] = \begin{cases} - \mu_0, & - x_{20} (\theta - t) < y_2 \\ \mu_0, & y_2 < - x_{20} (\theta - t) \end{cases}$$

and is determined from the maximum condition for $s^{\circ T} (0, \mu)^T$. Combining results (1.22) and (1.23), we obtain the strategy

$$M^{(e)} (t, y) = \begin{cases} - \mu_0, & H_+ < y_2 \\ \{\mu \mid |\mu| \leqslant \mu_0\}, & H_- < y_2 < H_+ \\ \mu_0, & y_2 < H_- \end{cases} \tag{1.24}$$

which solves Problem C_1. Completing its determination up to instant θ by the formula (1.12), we obtain the extremal strategy for Problem B_1.

We now construct the strategy for solving the original problem A_1. For this we transform the variables in strategy (1.24) with the use of formulas (1.10). We have

$$M^{(e)}(t,x) = \begin{cases} -\mu_0, & H_+ < x_1 \\ \{\mu \mid |\mu| \leqslant \mu_0\}, & H_- < x_1 < H_+ \\ \mu_0 & x_1 < H_- \end{cases} \tag{1.25}$$

Hence $M^{(H)}(t, x) = M^{(e)}(t, x)$ for $t < \theta$ and $M^{(H)}(t, x) = \mu^{(H)}(\theta)$ for $t = \theta$.

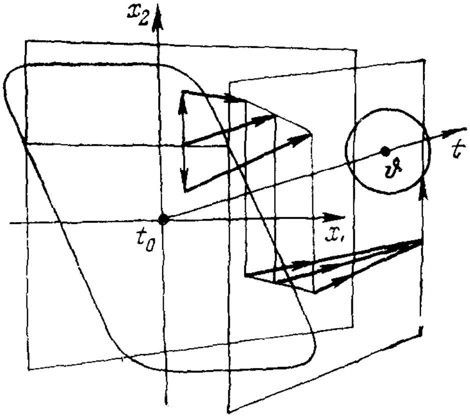

Fig. 1

Strategy (1.25) corresponds to the bridge

$$H_- < x_1 < H_+ \tag{1.26}$$

Bridge (1.26) depends upon the initial position. The transition from the strategy $M^{(H)}(t, x)$ to control u, which is effected by differentiating (1.6), automatically formalizes the impulse-sliding modes arising at the boundary of the bridge (1.26).

The region of initial conditions which lead to a successful completion of the first player's point of view, is the intersection of sets $W(\theta, t_0)$ and (1.20). If $\nu_0 \leqslant \mu_0$, then (1.20) holds and the considered region consists of the regions

$$\begin{cases} \mu_0 \leqslant x_{20} \leqslant \mu_0 + \varepsilon - \nu_0 \\ -h_+(x_{20}) + (\nu_0 - \mu_0 - x_{20})(\theta - t_0) \leqslant x_{10} \leqslant h_+(x_{20}) + \\ \quad (\mu_0 - \nu_0 - x_{20})(\theta - t_0) \end{cases} \tag{1.27}$$

$$\begin{cases} |x_{20}| \leqslant \mu_0 \\ -\sqrt{\varepsilon^2 - \nu_0^2} + (\nu_0 - \mu_0 - x_{20})(\theta - t_0) \leqslant x_{10} \leqslant \sqrt{\varepsilon^2 - \nu_0^2} + \\ \quad (\mu_0 - \nu_0 - x_{20})(\theta - t_0) \end{cases}$$

$$\begin{cases} \nu_0 - \mu_0 - \varepsilon \leqslant x_{20} < -\mu_0 \\ -h_-(x_{20}) + (\nu_0 - \mu_0 - x_{20})(\theta - t_0) \leqslant x_{10} \leqslant h_-(x_{20}) + \\ \quad (\mu_0 - \nu_0 - x_{20})(\theta - t_0) \end{cases}$$

Region (1.27) and bridge (1.26) are shown in Fig. 1.

When $\mu_0 < \nu_0$, requirement (1.20) yields the condition $\nu_0 - \sqrt{\varepsilon^2 - \nu_0^2}\,(\theta - t_0)^{-1} < \mu_0$ and constricts region (1.27) to the form resulting from (1.27) by replacing the ε occurring in the upper relations in the first and last regions by $[\varepsilon^2 - (\nu_0 - \mu_0)^2 (\theta - t_0)^2]^{1/2}$.

2. General case of a conflict-controlled linear system. We consider an object (2.1) and a target set (2.2)

$$x^{\cdot} = Ax + bu - cv = L(x, u, v) \tag{2.1}$$

$$d^2(x) = x^T Dx \leqslant \varepsilon^2 \tag{2.2}$$

Here A is an ($n \times n$)-matrix; b and c are n-vectors, and D is a nonnegative symmetric matrix. It is assumed that control u is restricted by constraint (1.4), while control v is subject to requirement (1.5). The players' admissible strategies are to be understood in the meaning of Sect. 1 with the addition that at the instant t the first player is informed about his own past history: $\{x(\tau)\}_{t_0 \leqslant \tau \leqslant t}$.

Problem A_2. For a given initial position (t_0, x_0) find the strategy $M^{(H)}$ which for any choice of admissible strategy N ensures that the phase point of system (2.1) reaches set (2.2) at instant θ.

Let us construct the auxiliary reduction problem. The system (2.1) under transforms has the form

$$\frac{d}{dt} x^* = L(x^*, \mu, \nu) + x_0 \chi(t - t_0) \tag{2.3}$$

If $\det A \neq 0$, the reaction of object (2,3) admits of a decomposition into the components

$$x^* = y - A^{-1}x_0, \quad t_0 < t \tag{2.4}$$

In (2.4) y is the solution of the Cauchy problem

$$y^{\cdot} = L(y, \mu, \nu), \quad y(t_0) = A^{-1}x_0 = y_0 \tag{2.5}$$

From (2.4), (2.1) and (2.5) follows the formula for the connection between variables

$$x = L(y, \mu, \nu), \quad t_0 < t \tag{2.6}$$

Thus, Problem A_2 is equivalent to the following problem.

Problem B_2. For a given initial position (t_0, y_0) find the strategy $M^{(H)}$ which for any choice of admissible strategy N ensures that the phase point of system (2.5) reaches at the instant θ the set (2.2), where $x = L[y, \mu(\theta), \nu(\theta)]$.

The value of $\mu^{(H)}(\theta)$ constituing set $M^{(H)}(\theta, y)$ must solve the problem

$$\min_{\mu(\theta)} \max_{\nu(\theta)} d(x) = d(x^\circ), \quad x^\circ = L[y, \mu^{(H)}(\theta), \nu^{(H)}(\theta)] \tag{2.7}$$
$$d(x^\circ) \leqslant \varepsilon$$

where x is defined by formula (2.6) $(t = \theta)$. The maximization in (2.7) yields

$$\nu^\circ(\theta) = -\nu_0 \operatorname{sign}\ [g_c^T y + \gamma\mu(\theta)]; \quad g_c = A^T Dc, \quad \gamma = b^T Dc \tag{2.8}$$

We consider two cases.

First let $\gamma = 0$. Then according to (2.8)

$$\nu^{(H)}(\theta) = -\nu_0 \operatorname{sign}\ g_c^T y \tag{2.9}$$

since $\nu^\circ(\theta)$ is independent of $\mu(\theta)$. With allowance for (2.9) we can obtain

$$\mu^{(H)}(\theta) = \begin{cases} \mu_0, & g_b^T y < -\mu_0\alpha \\ \mu_1(\theta), & |g_b^T y| \leqslant \mu_0\alpha \\ -\mu_0, & \mu_0\alpha < g_b^T y \end{cases} \tag{2.10}$$

$$g_b = A^T Db, \quad \alpha = d^2(b), \quad \mu_1(\theta) = \alpha^{-1} g_b^T y$$

Now let $\gamma > 0$. The minimization in (2.7) with condition (2.8) yields

$$\mu^{(H)}(\theta) = \begin{cases} \mu_0, & \begin{cases} \gamma\nu_0 < r^T y \text{ and } g_b{}^T y < \gamma\nu_0 - \alpha\mu_0 \\ |r^T y| < \gamma\nu_0 \text{ and } g_c{}^T y < -\gamma\mu_0 \\ r^T y < -\gamma\nu_0 \text{ and } g_b{}^T y < -\gamma\nu_0 - \alpha\mu_0 \end{cases} \\ \alpha^{-1}\gamma\nu_0 - \alpha^{-1}g_b{}^T y, & \gamma\nu_0 < r^T y \text{ and } \gamma\nu_0 - \alpha\mu_0 < g_b{}^T y < \gamma\nu_0 + \alpha\mu_0 \\ -\gamma^{-1}g_c{}^T y, & |r^T y| \leqslant \gamma\nu_0 \text{ and } |g_c{}^T y| < \gamma\mu_0 \\ -\alpha^{-1}\gamma\nu_0 - \alpha^{-1}g_b{}^T y, & r^T y < -\gamma\nu_0 \text{ and } -\gamma\nu_0 - \alpha\mu_0 < g_b{}^T y < -\gamma\nu_0 + \alpha\mu_0 \\ -\mu_0, & \begin{cases} \gamma\nu_0 < r^T y \text{ and } \gamma\nu_0 + \alpha\mu_0 < g_b{}^T y \\ |r^T y| \leqslant \gamma\nu_0 \text{ and } \gamma\mu_0 < g_c{}^T y \\ r^T y < -\gamma\nu_0 \text{ and } -\gamma\nu_0 + \alpha\mu_0 < g_b{}^T y \end{cases} \end{cases} \quad (2.11)$$

In (2.11) we have adopted the notation $r = g_b - \alpha\gamma^{-1}g_c$. In accordance with (2.9)

$$\nu^{(H)}(\theta) = -\operatorname{sign}[g_c{}^T y + \gamma\mu^{(H)}(\theta)] \quad (2.12)$$

corresponds to Eq. (2.11).

Thus, to solve Problem B_2 it is sufficient to solve the following problem.

Problem C_2. For a given initial position (t_0, y_0) find the strategy $\mathrm{M}^{(H)}$ which for any choice of admissible strategy N would ensure that the phase point of system (2.5) reaches at the instant θ the set defined by the second of relations (2.7).

We solve this problem by the scheme in [1]. The set $W(\theta, t)$ of program absorption of the target is determined as the set of vectors w which satisfy the condition

$$\rho_2(s, t, \theta) - \rho_1(s, t, \theta) - \rho_{-\mathrm{M}}(s) - s^T e^{A(\theta-t)} w \leqslant 0 \quad (2.13)$$

for all unit vectors s. In (2.13) ρ_i is the support function of the attainability set of the i th player ($i = 1, 2$) and $\rho_{-\mathrm{M}}$ is the support function of set $-\mathrm{M}$ (M is the set of x° from (2.7)). We have

$$\rho_1 = \mu_0 \int_t^\theta |s^T e^{A(\theta-\tau)} b|\, d\tau, \quad \rho_2 = \nu_0 \int_t^\theta |s^T e^{A(\theta-\tau)} c|\, d\tau$$

Then, having defined the function $\varkappa(t, y) = \max_{\|s\| \leqslant 1} [s^T y - \rho_{W(\theta,t)}]$ and, from the maximum condition for $s^{\circ T} b\mu$, the set

$$\mathrm{M}^{(e)}(t, s^\circ) = \begin{cases} -\mu_0 \operatorname{sign} s^{\circ T} b, & s^{\circ T} b \neq 0 \\ \{\mu \mid |\mu| \leqslant \mu_0\}, & s^{\circ T} b = 0 \end{cases} \quad (2.14)$$

we obtain the strategy

$$\mathrm{M}^{(e)}(t, y) = \begin{cases} \{\mu \mid |\mu| \leqslant \mu_0\}, & \varkappa(t, y) \leqslant 0 \\ \mathrm{M}^{(e)}(t, s^\circ), & \varkappa(t, y) > 0 \end{cases}$$

which solves Problem C_2. In (2.14) $s^\circ(t, y)$ maximizes the left-hand part of (2.13). Defining the strategy $\mathrm{M}^{(H)}(t, y)$ at instant $t = \theta$ by formula (2.10), we arrive at the extremal strategy for Problem B_2. If in the latter we make the change

$$y = A^{-1}x_0 + \int_{t_0}^{t} x \, d\tau$$

which follows from (2.4), we obtain the strategy for solving the original Problem A_2. Now, in contrast to Sect. 1, the first player generally needs to know his own past history when making a decision.

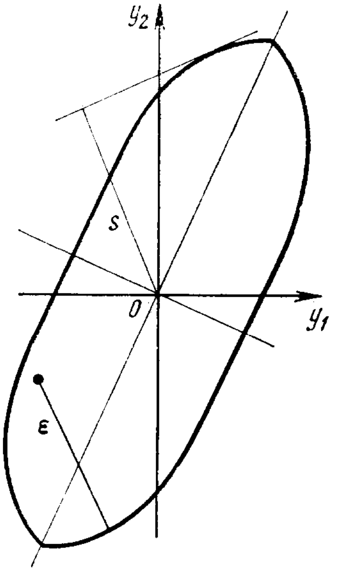

Fig. 2

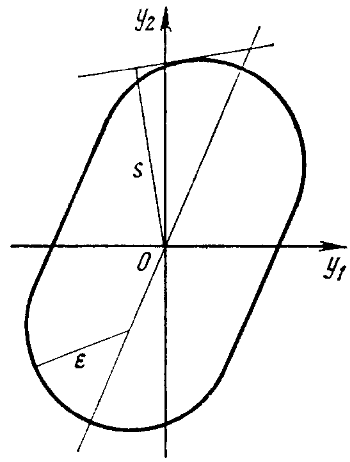

Fig. 3

Application. Let D be the unit ($n \times n$)-matrix, $\| b \| = \| c \| = 1$. We present the support function of set $-M$ for two cases: $b^T c = 0$ and $b = c$. We note that $-M = M$ and perform the transformation $z = Ay$. Then

$$\rho_M(s) = \| l \| \rho_{AM}(l \| l \|^{-1}), \; l = A^{-1T} s$$

The problem reduces to the calculation of the support function of set AM. If $b^T c = 0$, $\| l \| = 1$, $l^T b \geqslant 0$ and $l^T c \geqslant 0$, then $\rho_{AM}(l) = \varepsilon + l^T (b\mu_0 - c\nu_0)$. The value of $\rho_{AM}(l)$ for the other regions is found from the symmetry of set AM (Fig. 2). The set AM is shown in Fig. 3 for $b = c$. The necessary reduction condition is defined by inequality (1.15) and $\rho_{AM}(l) = \varepsilon + (\mu_0 - \nu_0) \, | \, l^T b \, |$.

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